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BOOK REVIEW

Benoît Perthame: Transport Equations in Biology

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Transport Equations in Biology is a compilation of lecture notes based on several courses and lectures developed by the author for an audience of mathematicians. As stated in the preface, the author's primary motivation is the mathematical study of partial differential equations intended to describe complex biological phenomena. True to his word the author's presentation of the material is a sequence of theorems and lemmas followed by proofs, as suitable for mathematicians, but a format that physicists and biologists rarely find attractive. If that were all there is to the book I could end the review here, but there is a great deal more.

The book begins with a chapter on the more familiar mathematical models of biological phenomena including epidemiology, prey-predator systems, nerve conduction, ecological competition and more. This first chapter establishes a level of mathematical rigor in the brief discussion of each of these phenomena so the reader can understand the domain of validity of the solutions. This is followed by a chapter on adaptive dynamics, that being phenotypic evolution driven by rare mutations during replication. The chapter presents asymptotic analysis of ordinary differential equations, using the Hamilton-Jacobi equations, in which the selection of small but frequent mutations is mathematically assessed. In the third chapter the author introduces the notion of *generalized relative entropy* in which the renormalized equations are driven to a steady state minimizing the entropy. This concept is subtle and is developed in the context of the epidemiologic renewal equation only to be generalized in a later chapter. Population balance equations in which cell division is described is taken up in the fourth chapter, where the generalized relative entropy is used to establish some universal properties of mitosis. The fifth chapter is the longest and deals with cell motion and the mobile reaction of cells to external stimulation. Much of this chapter is devoted to a mathematical discussion of the Keller-Segel model of chemotaxis in which a density of bacteria moves by means of diffusion and linear response to a chemical gradient. This chapter exemplifies the author's philosophy: "… the mathematical proof should be close to

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the ‘natural’ structure of the model and reflect somehow its meaning in terms of applications.” In this chapter I believe the author achieved his goal, particularly in the discussion of the kinetic theory of chemotaxis. The general mathematical tools used throughout the book are pulled together into the final chapter with some discussion given of the mathematical research on which the formalisms used in the book are based.

Although not encyclopedic in the scope of biological models discussed the author’s selection of phenomena is very broad and anyone studying mathematical biology will find much of value in both the mathematics presented and the rationale for how mathematical models are developed to describe complex phenomena. I would recommend this book to the serious student and researcher in mathematical biology, but I would also caution them that it will require both time and effort to master the material. But then so does everything else in life that is worthwhile.